Waiting time distributions for non-interacting fermions on a tight-binding chain

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Introduction

The concept of waiting time distribution (WTD) is a means to characterize the short time behavior of stochastic processes. It provides the probability distribution of waiting times between specific events and has recently been considered in electronics [1-5]. In this work we present a novel approach to WTD based on a 1D lattice model which constitutes a starting point to treating interacting systems [6].

Formalism

Central quantity: idle time probability (ITP) \( \Pi(t, \tau) \), probability that no event occurs in time interval \( I = [t_0, t_0 + \tau] \).

- In stationary state: ITP independent of \( t_0 \), WTD given by [3]:
  \[ W(\tau) = \langle \frac{d}{d\tau} \Pi(\tau) \rangle. \] (1)
- \( \Pi(\tau) = \langle 1 - \hat{Q}_c \rangle \) where \( \hat{Q}_c \) detects events during \( I \).
- With linear dispersion \( \varepsilon_0 \), \( \hat{Q}_c \) can be written as a projector on the space interval \([x_0, x_0 + \varepsilon_0] \).
- For non-interacting particles \( \Pi(\tau) = \det (1 - \hat{Q}_c) \) where \( \hat{Q}_c \) is a matrix of single-particle expectation values of \( \hat{Q}_c \) in a single particle basis.

Tight-binding approach

We describe the system scatterer + leads in a tight-binding formalism in the spirit of [7]:

\[ H = H_{\text{leads}} + H_{\text{scatterer}} + H_{\text{interactions}}, \]
\[ H_{\text{leads}} = -i \sum_{\alpha=\uparrow,\downarrow} \sum_{m=1}^{N_{\uparrow,\downarrow} + 1} [\varepsilon_{\alpha,m} \hat{a}_{\alpha,m} + \text{c.c.}] + \text{h.c.}. \]

- Initially the scatterer is empty and decoupled from the leads. \( N \) states around the band center in the left lead are occupied while the right lead is empty.

Dispersion \( \varepsilon_0 \) of \( H_{\text{leads}} \) (thick line) and linear approximation (thin line). Horizontal lines are the eigenenergies in the left and right lead. Blue (gray) states are initially occupied (empty).

- At \( t = 0 \), \( H_{\text{leads}} \) is turned on and particles propagate from left to right.
- During a finite time window the current becomes quasi-stationary. For \( I_t \), in this regime we can then use Eq. (1).
- For states around the band center the dispersion is linear. \( \hat{Q}_c \) thus reads
  \[ \hat{Q}_c = \sum_{m=1}^{N_{\uparrow,\downarrow}} \hat{a}_{\alpha,m} \hat{a}^\dagger_{\alpha,m} [\varepsilon_{\alpha,m} - \varepsilon_0]. \]

Quantum point contact

For non-interacting particles \( \Pi(\tau) \) is turned on and particles propagate from left to right.

Time dependent current for a QPC at varying transmission.

WTD for a QPC, data points: tight-binding approach, solid lines: scattering approach.

- For \( t_{\text{QPC}} = t \), \( W(\tau) \) is a Wigner-Dyson distribution.
- For small \( t_{\text{QPC}} \) the WTD approaches an exponential form while the Pauli-suppression at \( \tau = 0 \) persists.

Serial double quantum dot

WTD for a serial DQD with varying inter-dot coupling strength \( t_0 \).

Parallel double quantum dot

Parallel DQD with magnetic flux \( \Phi \) penetrating the area between the dots. Particles acquire a phase factor \( e^{i \Phi/\Phi_0} \) when hopping (counter) clockwise between the leads and the dots (red and blue arrows), where
  \[ \phi = 2 \pi (\Phi/\Phi_0) = (\pi/2)/\Phi_0. \] Direct hopping between the dots is phase-neutral (green arrow).

Single level

For a single level an analytic expression for the WTD can be obtained in the high bias limit [1] :

\[ W(\tau) = \frac{\Gamma_0}{\Gamma_L + \Gamma_R} \left( e^{-\Gamma_L \tau} - e^{-\Gamma_R \tau} \right)^{\frac{\Gamma_0}{\Gamma_L + \Gamma_R}} \left( e^{\Gamma_L \tau} - e^{\Gamma_R \tau} \right)^{\frac{\Gamma_0}{\Gamma_L + \Gamma_R}} \gamma^2 \tau^2 e^{-\gamma^2 \tau^2}, \] (2)

\[ \Gamma_0 = 4t_0^2 L/R \gamma. \quad \alpha = L, R. \]

In our model the bias voltage is given by the width of the initially occupied window of states. Typically \( \tau V = 2 \alpha \), so that Eq. (2) is valid in the limit \( t_{\text{QPC}}, t_0 \ll |\gamma| \ll 1 \).

Conclusions

- We have developed a numerical method to calculate the WTD for non-interacting fermions on a tight-binding chain.
- Our results agree well with the scattering approach as well as with the generalized master equation approach in the high bias regime.
- Our method can be extend to include interactions using time dependent DMRG.

References